

## Global CAPE Model Optimization

Adam Butler, CFA  
Michael Philbrick  
Rodrigo Gordillo  
*Darwin Funds*  
Phone: 416.572.5474  
Email: [evolve@darwinfunds.ca](mailto:evolve@darwinfunds.ca)  
Web: [www.darwinfunds.ca](http://www.darwinfunds.ca)

In collaboration with  
Mebane Faber  
*Cambria Quantitative Research*  
Phone: 310.683.5500  
Email: [info@cambriainvestments.com](mailto:info@cambriainvestments.com)  
Web: [www.cambriainvestments.com](http://www.cambriainvestments.com)

October 16, 2012

### Abstract

We use the Shiller CAPE Model proposed by Mebane Faber as a template for the exploration of a variety of portfolio optimization methods. By virtue of the Model's systematic allocation to the 'cheapest' markets with the highest theoretical risk premia, the model has the potential to extract high costs from 'behavioural taxes' related to the model's extreme volatility and drawdown character. We apply several portfolio optimization techniques with the objective of maximizing portfolio Sharpe ratios, including dynamic volatility weighting, risk parity, target risk and minimum variance. Consistent with recent published research on robust portfolio optimization, return to risk ratios improve broadly, with the greatest impact achieved from procedures that manage positions and/or portfolios to an ex ante target volatility. A theoretical framework is also proposed.

A Leap of Faith.....	3
High Volatility Results in a Lower P/E: A Conceptual Framework.....	4
Benchmarks.....	6
Volatility Management .....	7
<i>Case 1. Equal Volatility Weight.....</i>	<i>7</i>
<i>Case 2. Volatility Budgets .....</i>	<i>8</i>
<i>Case 3. Equal Weight with a Portfolio Volatility Target.....</i>	<i>10</i>
<i>Case 4. Robust Risk Parity .....</i>	<i>12</i>
<i>Case 5. Minimum Variance.....</i>	<i>13</i>
<i>Case 6. Minimum Variance with Volatility Target.....</i>	<i>14</i>
Conclusion.....	15

## A Leap of Faith

The Paper '[Global Value: Building Trading Models with the 10-Year CAPE](#)' by Mebane Faber presented a system for screening global markets by valuation and consistently investing in the cheapest ones as measured by the Cyclically Adjusted P/E Ratio, or CAPE.

The raw strategy generated superb simulated returns over the period from 1980 through August 2012. By holding the bottom third of markets each month based on CAPE, an investor would have compounded his portfolio at 13.5% annualized vs. 9.4% annualized growth for an equal weight basket of available indices. However, as might be expected from a strategy that purchases markets when there is proverbial 'blood in the streets', that is when markets are in a severe state of distress, the volatility and drawdown profile of the strategy is quite extreme.

*In the paper, Faber quite rightly says, 'How many investors have the stomach to invest in these countries with potential for the markets to get even cheaper? How many professional investors would be willing to bear the career risk associated with being potentially wrong in buying these markets?'*

These are important questions for investors to ask, because the fact is that the markets that register as quantitatively most attractive from a value perspective at any given time are, almost by definition, the most feared, loathed, dangerous places to invest in the world – which is why they offer the highest long-term risk premiums.

We know from a variety of studies of investor behavior that investors find it very difficult to pull the trigger on investments when all of the news is negative, and everyone they know is scrambling to abandon those same investments as quickly as possible, and at any price. As a result, while investors may know cognitively that they should 'hold their nose' and buy the cheapest markets, when it comes right down to it most investors will chicken out.

The purpose of this paper is to examine a variety of ways to manage portfolio exposure to the cheapest markets in order to make it more palatable to pull the trigger on investments in these markets when they are most volatile and uncertain. We will also demonstrate that intelligent management of portfolio exposures to the cheapest markets results in lower portfolio volatility, lower drawdowns, and in many cases higher returns than the standard equal-weight strategy presented in the core paper.

## High Volatility Results in a Lower P/E: A Conceptual Framework

MBA, CFA and regulatory certification courses are chock full of models for discovering the intrinsic value of securities and markets on the basis of a wide variety of valuation metrics. The most theoretically coherent model – that is, the model with the most intuitive mathematical foundation – is the Gordon Growth Model, which derives the valuation of securities from inputs like expected returns, ROE, earnings growth, and payout ratios.

An integration of CAPM and the GGM links expected returns to the beta of a security and the risk free rate, so by implication these factors also impact the intrinsic value of a security according to the following equation:

$$P_0 = \frac{D_0 \times (1 + g)}{[R_f + \beta_i \times (E(R_m) - R_f)] - g}$$

where  $D_0$  is the current dividend,  $g$  is the expected rate of dividend growth,  $R_f$  is the risk-free rate,  $\beta_i$  is the beta of the security, and  $E(R_m)$  is the expected return of the market.

From this equation it is a simple step to derive the most commonly cited measure of value for a security, the Price / Earnings ratio by dividing both sides of the equation by observed earnings:

$$\frac{P_0}{E} = \frac{D_0/E \times (1 + g)}{[R_f + \beta_i \times (E(R_m) - R_f)] - g}$$

Astute readers will notice that the  $D_0/E$  ratio is what is commonly known as the dividend payout ratio, or the proportion of retained earnings that are paid out to shareholders as dividends. The reciprocal of this ratio is called the retention rate, and this is the proportion of retained earnings that are theoretically reinvested in the company to generate growth. Companies with a higher retention rate should theoretically grow more quickly than high dividend payers, so long as their ROE is greater than their cost of capital.

For those who are not so mathematically inclined, the equation indicates that the P/E ratio of a security is inversely proportional to the security's systematic risk, whereby securities that exhibit higher risk will be subject to a lower P/E ratio, and therefore a lower valuation. Further, and in support of the Fed model (which

incidentally has little in the way of supporting empirical evidence), the P/E ratio also rises as a function of a lower risk-free rate.

The P/E ratio attracts a great deal of attention in both academic and practitioner circles, and the ratio is commonly cited as a reason to expect high or low returns from potential equity investments. The ratio is also often calculated for the market in aggregate as a measure of whether a market is cheap or expensive.

By definition, the systematic risk of the market portfolio is equivalent to the volatility of the market portfolio. Therefore, the same equations may be generalized to markets so that markets that exhibit higher volatility should be assigned lower P/E ratios, while lower interest rates should promote a higher ratio.

Of course, the paper references a derivation of the P/E ratio called the Cyclically Adjusted Price Earnings Ratio, or CAPE, that was first proposed by Benjamin Graham and David Dodd in their seminal book, 'Security Analysis' in 1934, and which was eventually popularized by Robert Shiller in 1998. While the calculation of the CAPE is more complex than the traditional P/E ratio, it is conceptually similar, and it is theoretically subject to the same sensitivities to volatility and interest rates.

While market multiples are sensitive to both interest rates and volatility, this paper will focus on volatility. Specifically, we will hypothesize that higher market volatility implies a lower P/E ratio. Correspondingly, higher volatility will result in contracting market valuations that, all things equal, will lead to lower prices. On the other hand, if volatility is falling, markets should deliver a higher valuation multiple, which will manifest through higher prices.

$$\frac{P}{E} \propto \frac{1}{\sigma}$$

If we follow this logic, then an intuitive overlay to the traditional CAPE trading model might involve a mechanism that lowers exposure to markets as they exhibit higher volatility, and raises exposure when they demonstrate lower volatility.

The following case studies utilize daily total return data from MSCI for equity indices from 32 countries around the globe. We will examine the return and risk profiles of strategies which leverage the CAPE trading model described in the original paper, but which manage the volatility contributions of the individual constituents, and/or aggregate portfolio volatility, at each monthly rebalance date using a variety of methods. Unfortunately, MSCI only provides daily total return data

for markets going back to 1999; fortunately the 12 years since 1999 represent a very interesting environment for our investigation.

Importantly, the case studies presented below will not track the original CAPE Model results presented in the Faber paper exactly for two reasons:

- Portfolios in the Faber paper were rebalanced annually, while the approaches below are rebalanced quarterly or monthly, as noted.
- Risk metrics such as volatility and drawdown were cited at a calendar year frequency in the Faber paper, while this paper provides metrics at a daily frequency. This makes a very large difference, especially for drawdowns.

## Benchmarks

The benchmark for our studies will be the Shiller CAPE trading system as presented in the paper. However, we thought it would also be relevant to examine the performance of the S&P 500, MSCI ACWI, and an equal-weight basket of all markets over the period as well. Chart 1. and Table 1. provide the relevant context.

Chart 1. Cumulative total returns to four benchmarks (USD)

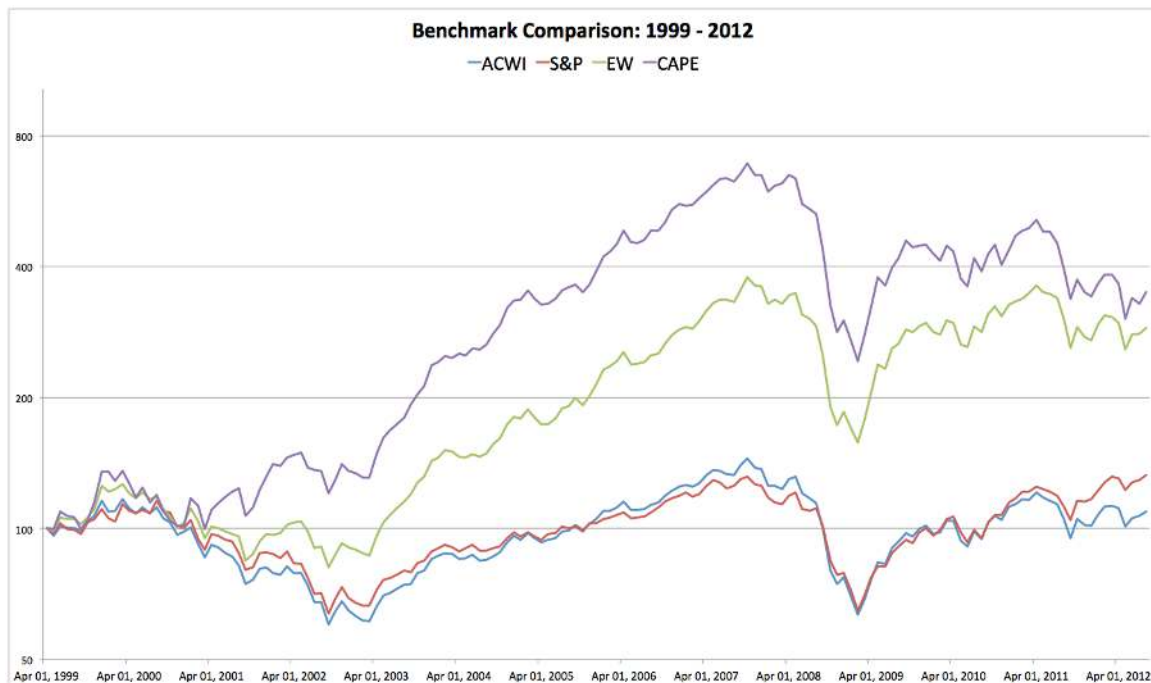


Table 1. Relevant total return statistics (USD)

Apr 1999 - Aug 2012	ACWI*	S&P 500	EW	CAPE
CAGR	0.66%	2.57%	8.39%	10.78%
STDEV	17.1%	21.5%	15.1%	19.3%
Sharpe	-0.09	0.01	0.41	0.44
MaxDD	-55.9%	-55.2%	-58.4%	-65.1%
% Positive Years	57.3%	66.0%	66.0%	69.0%

\*Monthly Data

Source: Faber, MSCI, Standard and Poors

Over the period studied, the equal weight basket dominated the capitalization weighted MSCI All-Cap World Index by a factor of almost 13x, largely because of the large overweight of a number of small emerging market economies in the equal weight index which did well during the inflationary growth phase of the mid-2000s.

## Volatility Management

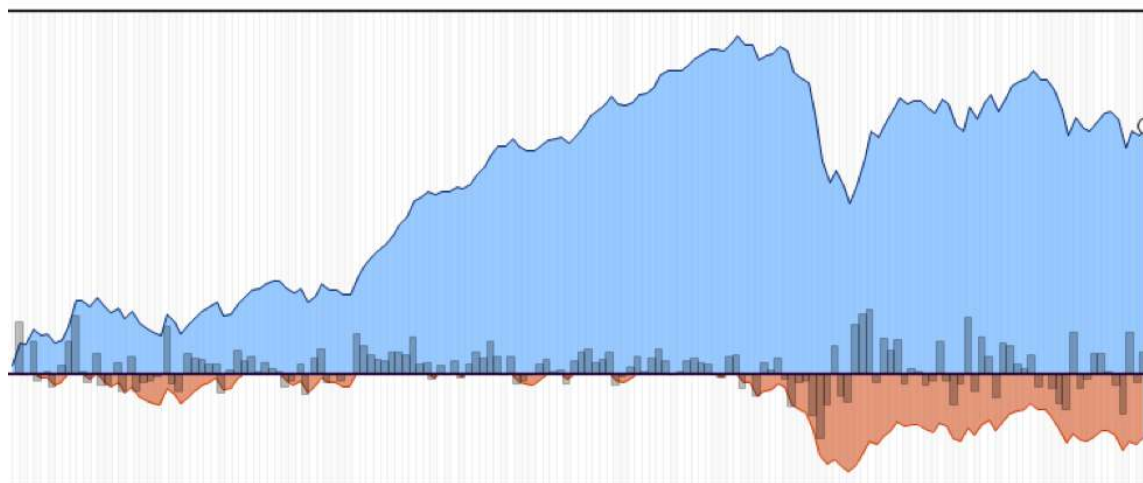
Our purpose is to investigate the impact of techniques applied to the universe of MSCI indices in order to manage the relative volatility contribution of each holding in the portfolio, and/or to manage the volatility of the portfolio itself. As discussed above, our hypothesis is that the market's valuation multiple should contract if estimated volatility increases, and expand as volatility contracts. If so, we should attempt to generate a volatility estimate so that we can vary exposure to markets as volatility expands and contracts.

### Case 1. Equal Volatility Weight

This simple overlay involves measuring the historical volatility of each holding in the index, and assembling the low CAPE portfolio each month so that each holding contributes an equal amount of volatility to the portfolio. In other words, with the traditional CAPE trading model each holding contributes an equal amount of capital to the model. With the equal volatility weight overlay, each holding contributes the same amount of estimated volatility to the portfolio. The portfolio is always fully invested.

Chart 2. CAPE trading model, equal volatility weight, fully invested

Date Range [19990401 to 20120831]



Curve		Trade		Time	
Total Return	323%	Trade Winning %	56%	% Winning Months	60%
CAGR	10.94%	Average Trade	1.17%	Average Winning Month	5.28%
Sharpe Ratio	0.46	Average Win	7.37%	Average Losing Month	-5.02%
Volatility	18.86%	Average Loss	-6.80%	Best Month	17.65%
MAR	0.17	Win/Loss Ratio	1.08	Worst Month	-25.39%
Max Daily Drawdown	-65.76%	Best Trade	72.3%	% Winning Years	79%
Average Drawdown	-1.73%	Worst Trade	-36.6%	Best Year	70.17%
Avg Drawdown Length	25.15	Avg Days in Trade	21.61	Worst Year	-52.91%
Avg Trades Per Year	90	Trades	1251	Positive 12 Month Periods	71%

Source: Faber, MSCI

This first overlay provides a marginal benefit vs. the raw CAPE strategy in terms of the Sharpe ratio, but it isn't very exciting. The drawdown profile is similar, as are the absolute returns. From our perspective, the lower volatility only matters if it reduces drawdowns, but this effect did not manifest in this case.

The challenge with this approach is that it is always fully invested. In 2008 when all global equity markets were dropping in concert, with almost perfect correlation, it did not help at all to balance risk equally between markets if the portfolio remained fully invested.

## Case 2. Volatility Budgets

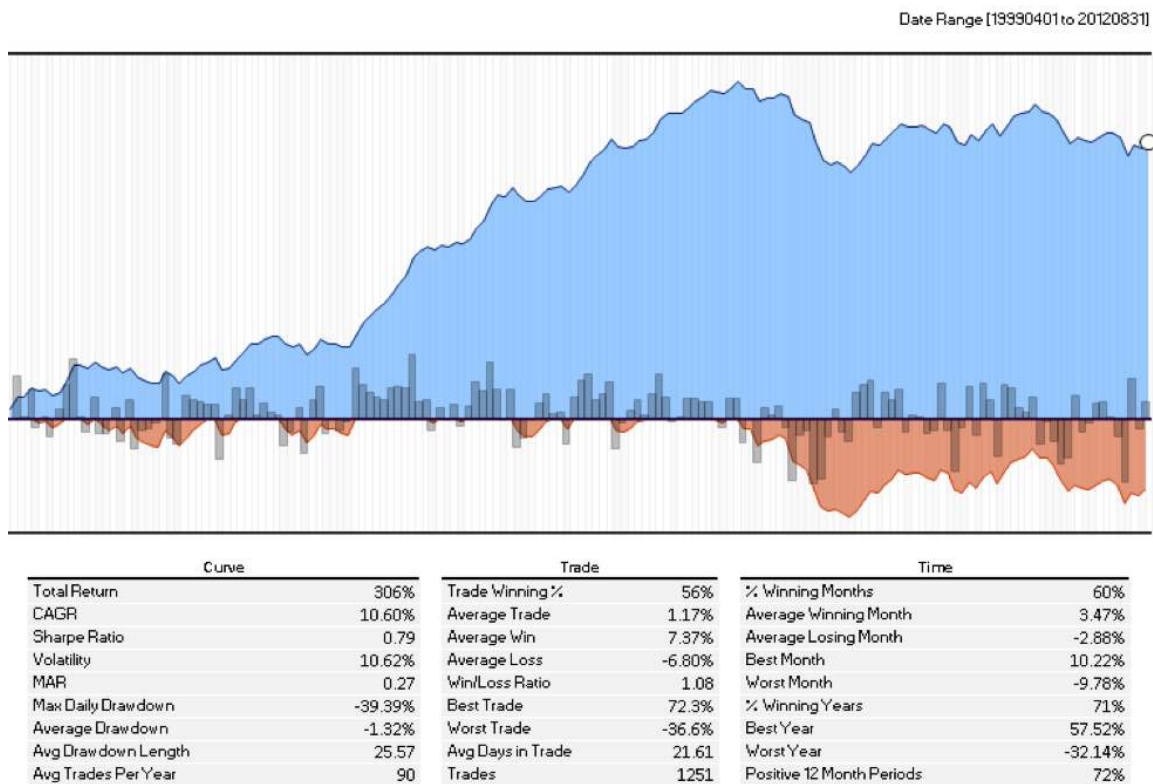
A slight variation on the equal volatility weight overlay is the application of volatility budgets for each index holding. In this case, each index contributes an equal amount of volatility to the portfolio *up to a fixed volatility target*. For example, each index is assigned the same volatility target, say 1% daily. If the portfolio has 10 holdings, then each holding should contribute a maximum of 1% daily volatility times its pro-rata share of the portfolio:  $1/10^{\text{th}}$ .



Imagine that on a rebalance date, the historical volatility (60 day) of one of the 10 low CAPE index holdings for the period is measured to be 1.25%, and the target volatility for each holding is set to *a maximum of 1%*. In this case, the overlay would allocate  $1\% / 1.25\%$ , or 80% of that holding's 10% pro-rata share, or 8% of the portfolio. The allocations for all 10 other holdings would be calculated in the same way.

If the sum of the individual allocations is less than 100%, the balance is held in cash. In this way, the total portfolio exposure is allowed to expand and contract over time as it adapts to the expansion and contraction in the volatility of the individual holdings.

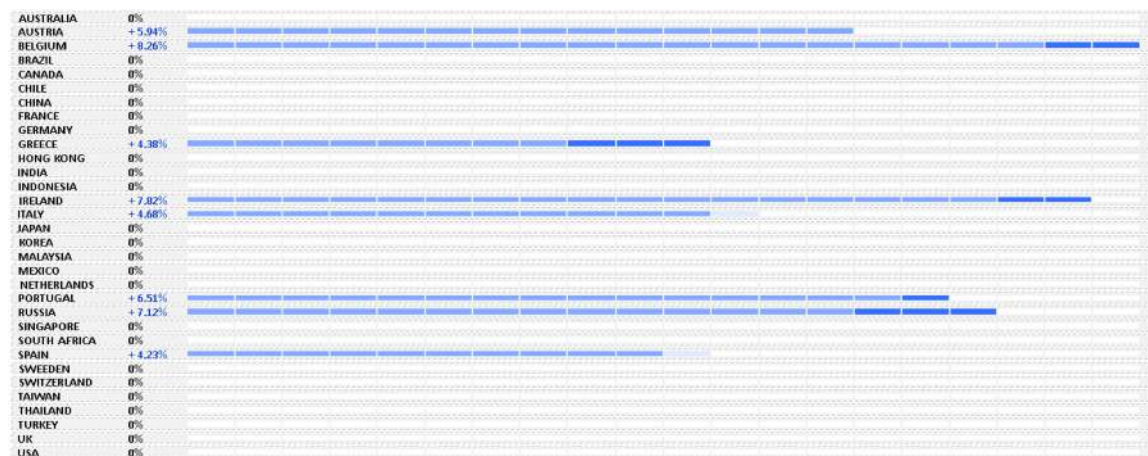
Chart 3. CAPE trading model, 1% daily holding volatility budget



Source: Faber, MSCI

For illustrative purposes, Chart 4. shows the theoretical allocations in this model as of the end of August, 2012. Note that the model is coming into September with total exposure of just 49%, which means it is 51% cash, strictly as a function of the budgets of the individual holdings for the month. In fact, over the full history of this approach, the average portfolio exposure was just 69%.

Chart 4. CAPE trading model, 1% daily holding volatility budget, holdings for Sep 2012



Source: Faber, MSCI

As a result of the portfolio's ability to adapt to the changing volatility of the individual low CAPE holdings, which lowers aggregate portfolio exposure during periods of volatile global contagion, this approach delivers almost twice as much return per unit of volatility (Sharpe .79 vs. 0.44 for the raw CAPE strategy), with 45% lower maximum drawdown. Further, this much lower risk profile is achieved with about the same absolute level of return (10.60% vs. 10.78% for the raw CAPE).

Our first two case studies managed volatility strictly at the level of the individual holdings. Our next cases will investigate the impact of managing volatility at the level of the overall portfolio.

### Case 3. Equal Weight with a Portfolio Volatility Target

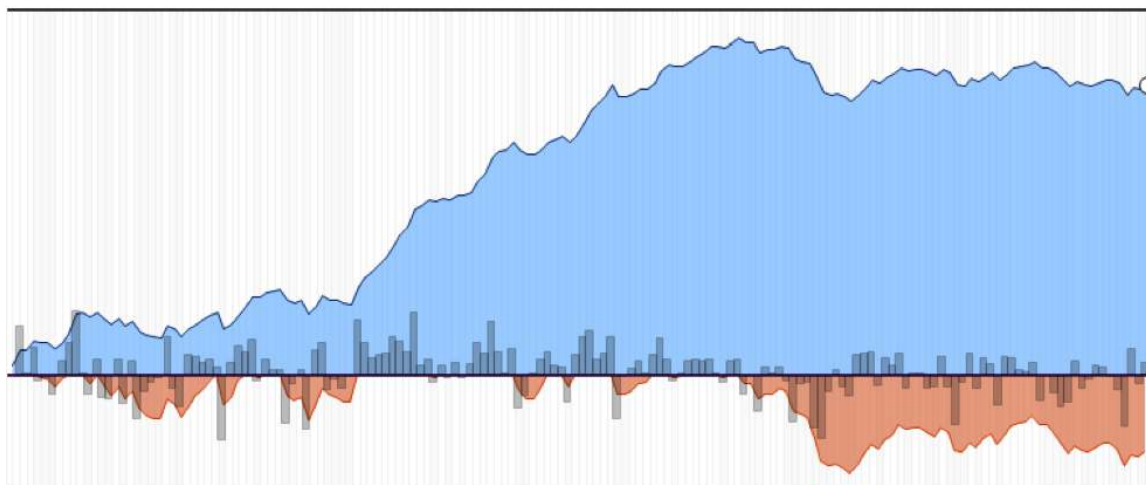
As discussed at length in the paper on [Adaptive Asset Allocation](#) (Butler & Philbrick, 2012), while volatility management at the individual security level will generally deliver similar returns with lower volatility and drawdowns than standard approaches, this technique misses some important information.

That is, the risk contribution of each holding in a portfolio is a function of the holding's individual volatility *as well as its covariance with the other holdings in the portfolio*. All things equal, if a holding has a low correlation with other portfolio constituents, it will lower the overall portfolio volatility. This dynamic is not captured if volatility is managed at the level of individual holdings. It must be managed at the overall portfolio level with an awareness of the covariance matrix.

The simplest example of this technique involves the traditional equally weighted basket of holdings. However, in this case the volatility of the portfolio of equally weighted holdings will be managed to a specific target at each monthly rebalance date. If the estimated volatility of the portfolio exceeds our target of 10% annualized (~2.9% monthly, 0.63% daily), portfolio exposure will be lowered accordingly in favor of cash. Note that the target is set to 10% annualized because this is the ex poste realized volatility of a typical 60/40 U.S. stock/bond portfolio.

Chart 5. CAPE model, equal weight, portfolio target volatility = 10% annualized

Date Range [19990401 to 20120831]



Curve		Trade		Time	
Total Return	360%	Trade Winning %	56%	% Winning Months	60%
CAGR	11.61%	Average Trade	1.17%	Average Winning Month	3.35%
Sharpe Ratio	1.01	Average Win	7.37%	Average Losing Month	-2.51%
Volatility	9.27%	Average Loss	-6.80%	Best Month	11.48%
MAR	0.38	Win/Loss Ratio	1.08	Worst Month	-8.47%
Max Daily Draw down	-30.72%	Best Trade	72.3%	% Winning Years	64%
Average Draw down	-1.48%	Worst Trade	-36.6%	Best Year	64.32%
Avg Draw down Length	27.91	Avg Days in Trade	21.61	Worst Year	-23.85%
Avg Trades Per Year	90	Trades	1251	Positive 12 Month Periods	68%

Source: Faber, MSCI

By managing the volatility of the portfolio itself, implicitly accounting for the covariance between the holdings as well as the volatility of the individual holdings, the approach delivered higher absolute returns, and almost 2.5x the Sharpe ratio relative to the raw CAPE model, with less than half the drawdown. This is a substantial improvement for a simple equal weight portfolio.

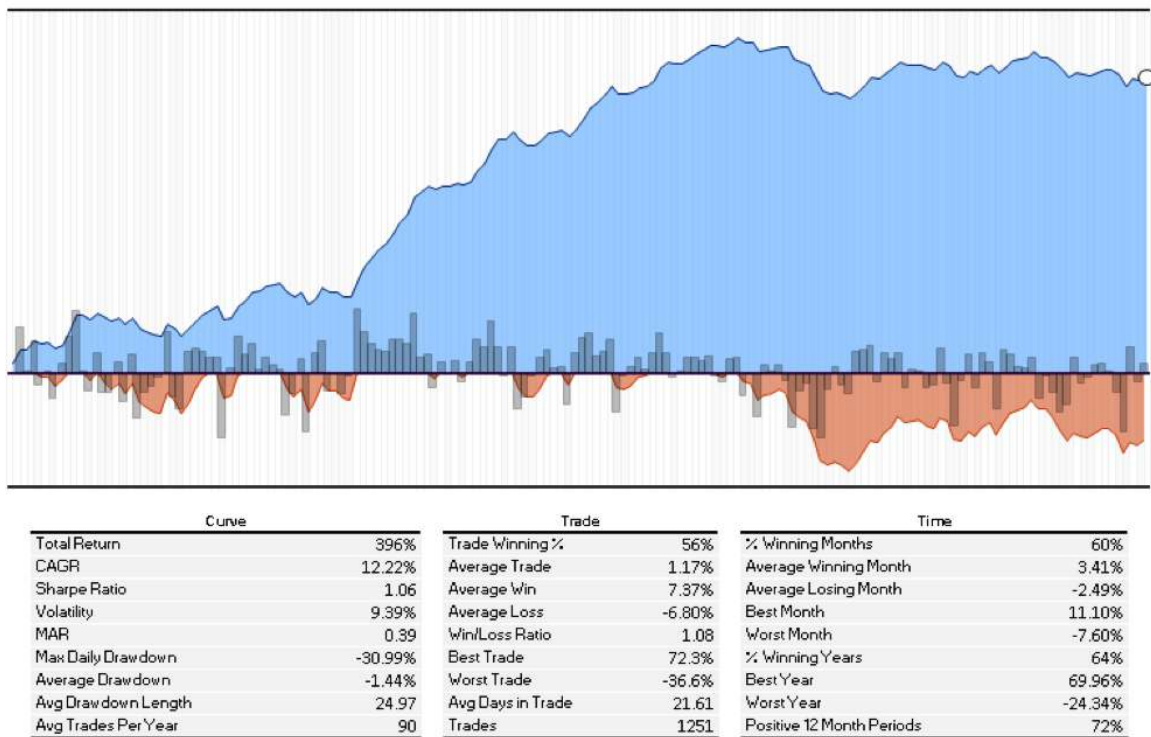
Note that while this approach targeted 10% portfolio volatility at each rebalance date based on trailing 60-day observations, the ex poste realized volatility of the strategy was just 9.27%, suggesting that the 60 day historical covariance matrix overestimated the volatility over the next month on average.

#### Case 4. Robust Risk Parity

No contemporary dynamic volatility paper is complete without an investigation of a risk parity approach. Our interpretation involves a combination of the techniques applied in Case 1. and Case 3, such that holdings are allocated based on equal volatility contributions rather than the equal capital contributions in Case 3, and then the equal risk portfolio is managed to a volatility target of 10%.

Chart 6. CAPE mode, robust risk parity, portfolio target volatility = 10% annualized

Date Range [19990401 to 20120831]



Source: Faber, MSCI

Consistent with what we found in case 1, there appears to be only a small advantage to allocating among the individual portfolio holdings based on volatility relative to traditional equal weight. All of the added value in this case seems to arise from the management of portfolio level volatility, not the equal allocation of risk among portfolio holdings.

Risk parity was conceived as a method of more effectively spreading risk across a basket of diversified asset classes, not allocating among constituents of a single asset class, and this investigation seems to validate this conception.

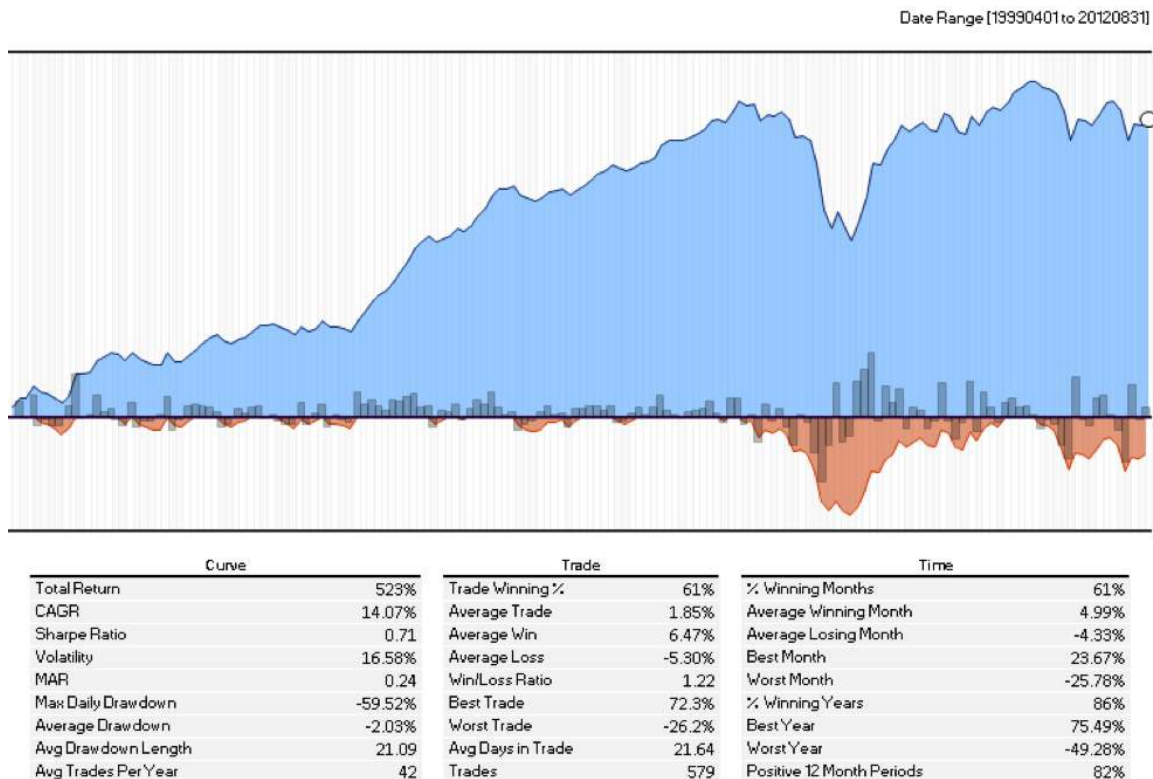
## Case 5. Minimum Variance

While the prior two cases accounted for the covariance between holdings by managing the volatility of the total portfolio, the objective was to hold all of the assets that meet the low CAPE criteria either in traditional equal weight, or equal volatility weight.

Minimum variance algorithms account for the covariance between assets by assembling portfolios with assets and weights that explicitly minimize portfolio volatility. Importantly, minimum variance algorithms do not usually hold all of the available assets in the portfolio. Rather, they select assets that help to achieve the objective of minimum variance via a combination of low volatility and low correlation. For this reason, a minimum variance overlay will almost certainly require less portfolio turnover.

For this case, we apply a minimum variance overlay to the low CAPE portfolio holdings at each monthly rebalance, but the portfolio is always fully invested.

Chart 7. CAPE model, minimum variance



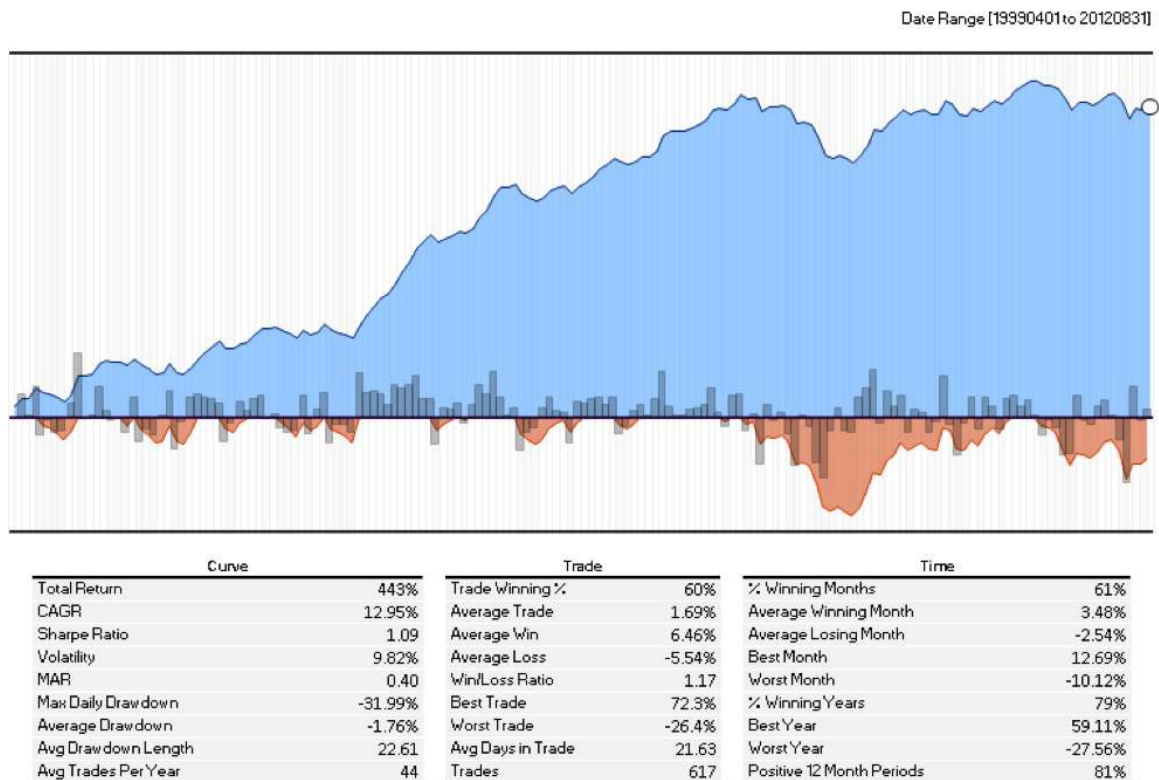
Source: Faber, MSCI

This is quite a boost in performance for a model that is always fully invested. While the drawdown profile does not decline materially, investors receive a big boost to absolute returns, and risk-adjusted returns almost double relative to the traditional CAPE model implementation.

#### Case 6. Minimum Variance with Volatility Target

Once the minimum variance portfolio is assembled each month, the volatility of the portfolio is estimated based on observations over the prior 60 days, and exposure is adjusted to target 10% portfolio volatility.

Chart 8. CAPE model, Minimum Variance, Portfolio Volatility Target = 10%



Source: Faber, MSCI

The minimum variance algorithm does not seem to add a great deal to portfolio performance relative to the other cases that manage portfolio level volatility (cases 3 and 4) - at first glance. There is a slight increase in annualized returns, with similar drawdowns and volatility versus the other volatility target methods. However, minimum variance achieves the same performance with half the number of trades. Further, the algorithm delivers positive returns over 81% of all rolling 12-month periods, versus about 70% for cases 3 and 4.



## Conclusion

The following table summarizes the progression of results from the case studies in this investigation. All tests below the CAPE test are risk-managed overlays on the raw CAPE system.

Global CAPE Model Overlay Summary					
1992 - 2012	CAGR	Volatility	Sharpe	MaxDD	% Positive Years
ACWI	0.66%	17.1%	0.04	-55.9%	57.3%
S&P 500	2.6%	21.5%	0.01	-55.2%	66.0%
Equal Weight	8.4%	15.1%	0.41	-58.4%	66.0%
CAPE	10.8%	19.3%	0.44	-65.1%	69.0%
Equal Volatility	10.9%	18.9%	0.46	-65.8%	71.0%
Individual Target Vol	10.6%	10.6%	0.79	-39.4%	72.0%
EW Portfolio Target Vol	11.6%	9.3%	1.01	-30.7%	68.0%
Risk Parity	12.2%	9.4%	1.06	-31.0%	72.0%
Minimum Variance	14.1%	16.6%	0.71	-59.5%	82.0%
MinVar Target Vol	13.0%	9.8%	1.09	32.0%	81.0%

Shaded rows indicate target volatility approaches

Source: MSCI, Faber

Several observations stand out. First of all, the equal weight basket of all MSCI markets outperformed both the S&P 500 and the MSCI All-Cap World Index by several orders of magnitude over the period. This is consistent with the findings of other empirical studies, which broadly suggest that that simple 1/n approaches dominate cap-weighted approaches on both absolute and risk adjusted return measures for most markets. It is important to note however, that in this case the equal weight basket places greater emphasis on very small markets, which might impose quite substantial liquidity constraints (and costs).

Secondly, the CAPE approach delivers measurably better returns than the equal weight basket, but not surprisingly at the expense of higher. After all, the CAPE model buys markets when they are in the throes of violent upheavals; it is the intense pressure of tumultuous periods that forges long-term market bottoms.

Thirdly, risk management overlays that require portfolios to always be fully invested offer lower risk-adjusted returns than overlays that allow portfolio exposure to expand and contract in response to the volatility of the individual holdings, or of the total portfolio. For example, the Equal Volatility overlay, which distributed volatility equally across CAPE holdings, but is always fully invested, delivered approximately the same absolute and risk-adjusted performance as the equal-weight CAPE model.

The Risk Parity and EW Portfolio Target Vol approaches however, which simply add an extra layer that targets portfolio volatility of 10% to the Equal Volatility and Equal Weight portfolios respectively, deliver higher absolute returns with about half the volatility.

Fourthly, Minimum Variance algorithms add very substantial value on both a risk adjusted and absolute basis. The Minimum Variance CAPE portfolio, for example, delivers fully 2.5% per year better returns than the raw CAPE strategy, with over 80% positive rolling 12-month periods.

Notably, the Minimum Variance portfolio has about half the turnover of the other strategies because it does not hold all of the low CAPE markets at each rebalance. Rather it creates portfolios of securities that deliver the lowest possible volatility out of all possible portfolio combinations.

One might speculate that one reason for this outperformance is that the Minimum Variance algorithm might choose cheap markets in non-correlated regions because it explicitly accounts for the covariance matrix, preferring diversification when all markets are equally volatile.

As expected, managing overall portfolio volatility substantially improves the risk-adjusted performance of the Minimum Variance CAPE portfolio, while still delivering the second highest absolute returns of all the approaches investigated.

The low CAPE model seeks to invest in the cheapest markets around the world because, theoretically and empirically, cheap markets imply higher expected returns. However, as our Adaptive Asset Allocation paper demonstrated with momentum as a return estimate, two more estimates are required to assemble optimal portfolios. We need estimates for each asset's volatility as well as the covariance between the assets.

When these estimates are integrated into the process of portfolio optimization, like in the Minimum Variance examples above, portfolio achieve substantially higher absolute and risk-adjusted returns. Portfolio volatility targets then serve to adjust the portfolio exposure to achieve the appropriate position on the Securities Market Line – that is, to achieve the maximum return possible for our target level of risk.



Butler Philbrick Gordillo is part of Macquarie Private Wealth Inc.

This material is provided for general information and is not to be construed as an offer or solicitation for the sale or purchase of securities mentioned herein. Past performance may not be repeated. Every effort has been made to compile this material from reliable sources however no warranty can be made as to its accuracy or completeness. The comments contained herein are general in nature and are not intended to be, nor should be construed to be, legal or tax advice to any particular individual. Accordingly, individuals should consult their own tax advisors for advice with respect to the tax consequences to them, having regard to their own particular circumstances. Before acting on any of the above, please seek individual financial advice based on your personal circumstances. However, neither the author or Macquarie Private Wealth Inc. (MPW) makes any representation or warranty, expressed or implied, in respect thereof, or takes any responsibility for any errors or omissions which may be contained herein or accepts any liability whatsoever for any loss arising from any use or reliance on this report or its contents. Macquarie Private Wealth Inc. is a member of Canadian Investor Protection Fund and IIROC. No entity within the Macquarie Group of Companies is registered as a bank or an authorized foreign bank in Canada under the Bank Act, S.C. 1991, c. 46 and no entity within the Macquarie Group of Companies is regulated in Canada as a financial institution, bank holding company or an insurance holding company. Macquarie Bank Limited ABN 46 008 583 542 (MBL) is a company incorporated in Australia and authorized under the Banking Act 1959 (Australia) to conduct banking business in Australia. MBL is not authorized to conduct business in Canada. No entity within the Macquarie Group of Companies other than MBL is an authorized deposit-taking institution for the purposes of the Banking Act 1959 (Australia), and their obligations do not represent deposits or other liabilities of MBL. MBL does not guarantee or otherwise provide assurance in respect of the obligations of any other Macquarie Group company.